UNIT I: SET, RELATIONS AND FUNCTIONS

SET THEORY

- A **Set** is a definite collection of objects, objects are known as elements or members of the set.
  An element 'a' belong to a set A can be written as '$a \in A$',
  '$a \notin A$' denotes that a is not an element of the set A.

- **Representation of a Set**
  A set can be represented by various methods. 3 common methods used for representing set:
  1. Roaster form or tabular form method.
  2. Set Builder method.

- **Roster form**
  In this representation, elements are listed within the pair of brackets {} and are separated by commas.

  For example:

1. Let N is the set of natural numbers less than 5.
   N = { 1 , 2 , 3, 4 }.
2. The set of all vowels in English alphabet.
   V = { a , e , i , o , u }.

- **Set builder form**
  In set builder set is describe by a property that its member must satisfy.

  For example:

  1. {x : x is even number divisible by 6 and less than 100}.
  2. {x : x is natural number less than 10}.

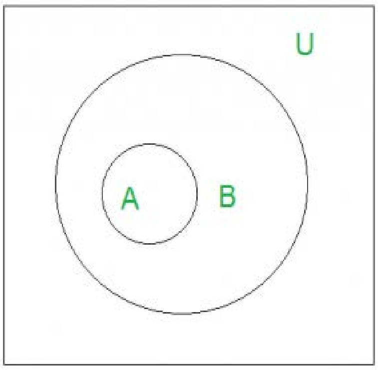
- **Equal sets**
  Two sets are said to be equal if both have same elements.

  For example A = {1, 3, 9, 7} and B = {3, 1, 7, 9} are equal sets.

NOTE: Order of elements of a set doesn't matter.

- **Subset:**

A set A is said to be **subset** of another set B if and only if every element of set A is also anelement of other set Denoted by '⊆'.
'A ⊆ B 'denotes A is a subset of B.



U' denotes the universal set.
Above Venn diagram shows that A is Subset of B.

Size of the set S is known as **Cardinality number**, denoted as |S|.

Example: Let A be a set of odd positive integers less than 10.
Solution: A = {1,3,5,7,9}, Cardinality of the set is 5, i.e.,|A| = 5.

Note: Cardinality of null set is 0.

- **Power Sets**
  Power set is the set all possible subset of the set S. Denoted by P(S).
  Example : What is the power set of {0,1,2}?

  Solution:
  {∅}, {0}, {1}, {2}, {0,1}, {0,2}, {1,2}, {0,1,2}.

  Note :

  - Empty set and set itself is also member of this set of subsets.
  - **Cardinality of power set** is 2^n, where n is number of element in a set.

Cartesian Products

Let A and B be two sets. Cartesian product of A and B is denoted by A × B, is the set of all ordered pairs (a,b), where a belong to A and B belong to B.

RELATIONS

Relation or Binary relation R from set A to B is a subset of AxB which can be defined as aRb <=> (a,b) € R <=> R(a,b).

For two distinct set, A and B with cardinalities m and n, the maximum cardinality of the relation R from A to B is mn

Domain and Range:

Given two sets A and B and Relation from A to B is R (a,b),

Domain is defined as the set= { a | (a,b) € R for some b in B}

Range is defined as the set = {b | (a,b) € R for some a in A}.

Types of Relation:

1.**Empty Relation:** A relation R on a set A is called Empty if the set A is empty set.

2.**Reflexive Relation:** A relation R on a set A is called reflexive if (a,a) € R for every element a € A .i.e. if set A = {a,b} then R = {(a,a), (b,b)} is reflexive relation.

3.**Symmetric Relation:** A relation R on a set A is called symmetric if (b,a) € R when (a,b) € R.
i.e. The relation R={(4,5),(5,4),(6,5),(5,6)} on set A={4,5,6} is symmetric.

4.Transiitive Relation: A relation R on a set A is called transitive if (a,b) € R and (b,c) € R then (a,c) € R
for all a,b,c € A.i.e. Relation R={(1,2),(2,3),(1,3)} on set A={1,2,3} is transitive.

5..**Equivalence Relation:** A relation is an Equivalence Relation if it is reflexive, symmetric, and transitive
The relation R={(1,1),(2,2),(3,3),(1,2),(2,1),(2,3),(3,2),(1,3),(3,1)} on set A={1,2,3} is equivalence relation as it is reflexive, symmetric, and transitive.

FUNCTIONS:

One-one function:(injective function)

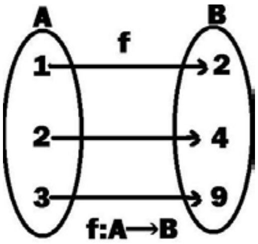
A Function is said to be a One-To-One Function if every element of Domain of the function have its own and unique element in Range of the Function.

A function from set A to set B is said to be a One-To-One Function if no two or more elements of set A have same same elements mapped or imaged in set B.

i.e.if $f:X \rightarrow Y$ then for every x_1, x_2 €X

if $f(x_1)=f(x_2)$ then $x_1=x_2$

example: The function $f:N \rightarrow N$ given by $f(x)=x^2$ is one -one



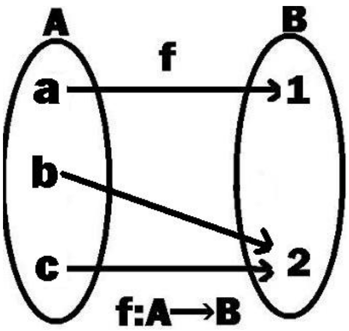
Onto function(surjective function)

A function f:X→Y is said to be onto function if every element of Y is an image of some element of X
i.e. for every y in Y there exists x in X such that f(x)=y

Example:

If set A= {a,b,c} and set B={1,2}

And “f” is a onto function such that f: A→B is defined by:



Bijective function:

A function f:X →Y is said to be bijective function if f is both one-one and onto function

Inverse function:

A function f:X→Y is said to be invertible if is both one-one and onto ,then f^{-1}:Y→X is said to be inverible function and f^{-1}(y)=x.

MATHEMATICAL LOGIC

Proposition:

A proposition is defined as a declarative sentence that is either True or False, but not both.
The **Truth Value** of a proposition is True(denoted as T) if it is a true statement, and False(denoted as F) if it is a false statement.
For Example,

1.Bangalore is in Karnataka

2.”f” is a vowel

3.sun rises in east and sets in west.

Logical connectives:

1. NEGATION:

If p is a proposition then ~p is called the negation.

the truth table for negation

p	~p
T	F
F	T

2.CONJUNCTION:

Two simple proposition p&q are connected by the connective “ and” ,the compound proposition is called conjuction,denoted by p^q

The truth table for p^q

p	q	p ∧ q
T	T	T
T	F	F
F	T	F
F	F	F

3.DISJUNCTION:

Two simple proposition p & q are connected by the connective " OR" ,the compound proposition thus obtained is called disjunction, denoted by $p \vee q$

p	**q**	**$p \vee q$**
T	T	T
T	F	T
F	T	T
F	F	F

4.IMPLICATION:(conditional)

Two simple proposition p and q are connected with the connective" if then" ,the resulted compound proposition thus obtained is called implication ,denoted by $p \rightarrow q$

p	**q**	**$p \rightarrow q$**
T	T	T
T	F	F
F	T	T
F	F	T

5.DOUBLE IMPLICATION:

Two simple proposition p and q are connected with the connective" if aand only if " ,the resulted compound proposition thus obtained is called double implication ,denoted by p ↔q

The truth table is

p	**q**	**p↔ q**
T	T	T
T	F	F
F	T	F
F	F	T