Unit 2: Measures of central tendency

1) Calculate the mean, median and mode from the following data. (April 2015)

Class interval	0-10	10-20	20-30	30-40	40-50	50-60
frequency	14	23	35	20	8	5

Ans:

c.I	f	mid value (x)	$d = x - A$ (A = 35)	$d' = d/c$ (c = 10)	fd'	c.f
0-10	14	5	-30	-3	-42	14
10-20	f_0 23	15	-20	-2	-46	37 m.
l 20-30	f_1 35 f	25	-10	-1	-35	72
30-40	f_2 20	35 = A	0	0	0	92
40-50	8	45	10	1	8	100
50-60	5	55	20	2	10	105
	n = 105				$\Sigma fd' = -105$	

Arithmetic mean

$$\bar{X} = A + \frac{\Sigma fd'}{n} \times c$$

$$= 35 + \frac{(-105)}{105} \times 10$$

$$= 35 - 10 = 25$$

Median

$$\frac{n^{th}}{2} \text{ term} = \frac{105}{2} = 52.5^{th} \text{ term}$$

$f = 35 \quad l = 20 \quad m = 37 \quad c = 10$

$$Me = l + \frac{n/2 - m}{f} \times c$$

$$= 20 + \frac{\frac{105}{2} - 37}{35} \times 10$$

$$= 20 + \frac{52.5 - 37}{35} \times 10$$

$$= 20 + 4.4285$$

$$= 24.4285$$

Mode:

$\Delta_1 = f_1 - f_0 = 35 - 23 = 12$

$\Delta_2 = f_1 - f_2 = 35 - 20 = 15$

$$Z = l + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times c$$

$$= 20 + \frac{12}{12 + 15} \times 10$$

$$= 20 + \frac{12}{27} \times 10$$

$$= 20 + 4.444 = 24.444$$

2) calculate the median from the following data (April 2015)

Less than	10	20	30	40	50	60	70	80
frequency	4	16	40	76	96	112	120	125

Ans: convert the x value in to C.I.

C.I	f	c.f
0-10	4	4
10-20	12	16
20-30	24	40 m
30-40	36 f	76 ←
40-50	20	96
50-60	16	112
60-70	8	120
70-80	5	125
	n=125	

$\frac{n}{2}^{th}$ term $= \frac{125}{2} = 62.5^{th}$ term

$$Me = l + \frac{n/2 - m}{f} \times c$$

$$= 30 + \frac{\frac{125}{2} - 40}{36} \times 10$$

$$= 30 + \frac{62.5 - 40}{36} \times 10$$

$$= 30 + 6.25 = 36.25$$

3) calculate arithmetic mean from the following (may 2014)

x	f
100 and above	50
200 & above	46
300 & above	36
400 & above	18
500 & above	6
600 & above	3
700 & above	1

Ans: convert the X value into class interval $A = 450$, $c = 100$

X	f	midvalue	$d = x - A$	$d' = d/c$	fd'
100-200	4	150	-300	-3	-12
200-300	10	250	-200	-2	-20
300-400	18	350	-100	-1	-18
400-500	14	450 = A	0	0	0
500-600	3	550	100	1	3
600-700	2	650	200	2	4
700-800	1	750	300	3	3
	$n = 50$				$\Sigma fd' = -23$

$$\bar{x} = A + \frac{\Sigma fd'}{n} \times c$$

$$= 450 + \frac{(-23)}{50} \times 100$$

$$= 450 - 46 = 404$$

4) Calculate A.M. from the following data (May 2017)

$A = 35$, $c = 10$

X	f	midvalue(x)	$d = x - A$	$d' = d/c$	fd'
0-10	2	5	-30	-3	-6
10-20	4	15	-20	-2	-8
20-30	6	25	-10	-1	-6
30-40	8	35	0	0	0
40-50	10	45	10	1	10
50-60	12	55	20	2	24
60-70	14	65	30	3	42
	$n = 56$				$\Sigma fd' = 56$

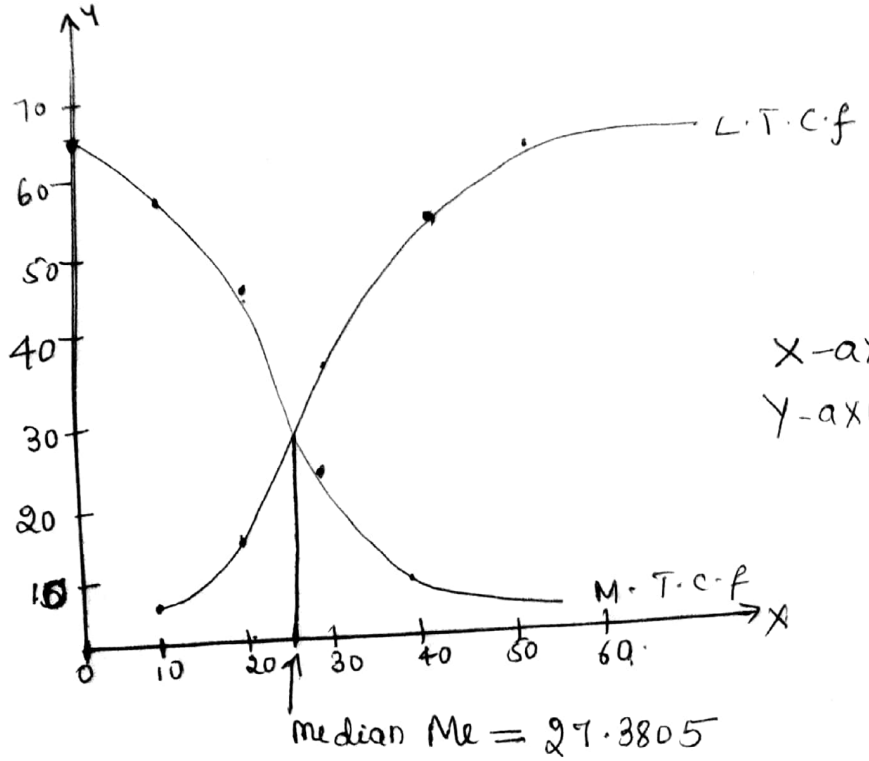
$$\bar{x} = A + \frac{\Sigma fd'}{n} \times c$$

$$= 35 + \frac{56}{56} \times (10)$$

$$= 35 + 10 = 45$$

5) Draw Dgive curves from the following data, and measure the median value. verify it by actual calculation (may 2017)

C.I	f	L.T.C.f	M.T.C.f
0-10	5	5	63
10-20	11	16 m	58
l 20-30	21 f	37 ←	47
30-40	16	53	26
40-50	10	63	10
	n = 63		



X-axis: 1 ~~unit~~ cm = 10 units
Y-axis = 1 cm = 10 units

$$Me = l + \frac{\frac{n}{2} - m}{f} \times c$$

$$\frac{n}{2} = \frac{63}{2} = 31.5^{th} \text{ term}$$

$$= 20 + \frac{63/2 - 16}{21} \times 10$$

$$= 20 + \frac{31.5 - 16}{21} \times 10$$

$$= 20 + \frac{15.5}{21} \times 10$$

$$= 20 + 7.3805 = \underline{27.3805}$$

Measures of dispersion

1) calculate upper quartile from the following data (may 2016)

Income : (less than) :	2000	4000	6000	8000	10000	10,000 & above
No. of persons :	25	65	150	225	241	257.

Ans:

X	f	c.f	
0 - 2000	25	25	
2000 - 4000	40	65	
4000 - 6000	85	150	m
l 6000 - 8000	75	f 225	← Q_3
8000 - 10000	16	241	
10000 - 12000	16	257	

$n = 257$

$$Q_3 = \frac{3n}{4} = \frac{3(257)}{4} = 192.75^{th} \text{ term}$$

$$Q_3 = l + \frac{\frac{3n}{4} - m}{f} \times c$$

$$= 6000 + \frac{\frac{3(192.75)}{4} - 150}{75} \times 2000$$

$$= 6000 + \frac{192.75 - 150}{75} \times 2000$$

$$= 6000 + 1140 = 7140$$

2) calculate median, upper quartile, lower quartile and quartile deviation from the following data (May 2016)

Income (less than)	500	1000	1500	2000	2500	3000	3500	4000	4500	5000
No. of employees:	08	26	42	54	79	83	110	132	146	150

Ans:

X	f	C.f	
0-500	8	08	
500-1000	18	26 m	
l 1000-1500	16 f	42	Q_1
1500-2000	12	54	
2000-2500	25	79 m	
l 2500-3000	4 f	83	← Me
3000-3500	27	110 m	
l 3500-4000	22 f	132	Q_3
4000-4500	14	146	
4500-5000	4	150	
	n=150		

$Q_1 = \frac{n}{4}^{th}$ term

$= \frac{150}{4} = 37.5^{th}$ term

$Q_1 = l + \frac{\frac{n}{4} - m}{f} \times C$

$= 1000 + \frac{37.5 - 26}{16} \times 500$

$= 1000 + 221.153845$

$= 1221.153845$

Median $= \frac{n}{2}^{th}$ term $= \frac{150}{2} = 75^{th}$ term

$Me = l + \frac{\frac{n}{2} - m}{f} \times C$

$= 2500 + \frac{75 - 79}{150} \times 500$

$= 2500 - \frac{4}{150} \times 500$

$= 2500 - 13.333$

$= 2486.6667$.

$Q_3 = \frac{3n}{4} = \frac{3(150)}{4} = 1125^{th}$ term

$Q_3 = l + \frac{\frac{3n}{4} - m}{f} \times C$

$= 3500 + \frac{112.5 - 110}{22} \times 500$

$= 3500 + \frac{2.5}{22} \times 500$

$= 3500 + 56.81818$

$= 3556.81818$

$Q.D = \frac{Q_3 - Q_1}{2} = \frac{3556.81818 - 1221.153845}{2}$

$= 1167.83217$

(3) Following are the marks obtained by two students A & B in 10 Tests of marks each. Find who is better in studies and who should get the prize?

Test	1	2	3	4	5	6	7	8	9	10
A	44	80	76	48	52	72	72	51	60	54
B	48	75	54	60	63	69	72	51	57	66

standard deviation for A

x	$d = x - \bar{x}$ (x−50)	d^2
44	−6	36
80	+30	900
76	+26	676
48	−2	4
52	+2	4
72	+22	484
51	+1	1
60	+10	100
54	+4	16
537	$d = 87$	2221

$\bar{x} = \frac{\Sigma x}{n} = \frac{537}{10} = 53.7 \cong 50$

$\sigma = \sqrt{\frac{\Sigma d^2}{n} - \left(\frac{\Sigma d}{n}\right)^2}$

$= \sqrt{\frac{2221}{10} - \left(\frac{87}{10}\right)^2}$

$= \sqrt{222.1 - 75.69}$

$= \sqrt{146.41}$

$= 12.1$

$C.V = \frac{\sigma}{\bar{x}} \times 100$

$= \frac{12.1}{53.7} \times 100$

$= 22.5325$

standard deviation for B

x	$d = x - A$ (x−60)	d^2
48	−12	144
75	+15	225
54	−6	36
60	0	0
63	+3	9
69	+9	81
72	+12	144
51	−9	81
57	−3	9
66	+6	36
$\Sigma x = 615$	$\Sigma d = 15$	$\Sigma d^2 = 765$

$\bar{x} = \frac{\Sigma x}{n} = \frac{615}{10} = 61.5 \cong 60$

$\sigma = \sqrt{\frac{\Sigma d^2}{n} - \left(\frac{\Sigma d}{n}\right)^2}$

$= \sqrt{\frac{765}{10} - \left(\frac{15}{10}\right)^2}$

$= \sqrt{76.5 - (1.5)^2}$

$= \sqrt{76.5 - 2.25}$

$= \sqrt{74.25} = 8.6168$

$C.V = \frac{\sigma}{\bar{x}} \times 100$

$= \frac{8.6168}{61.5} \times 100 = 14.0111$

$\therefore$ C.V is less in B. So B is Better than A

④ The following table gives the age distribution of boys and girls in a school. Find which group is more variable in age

Age	No. of boys	No. of girls
13	12	18
14	15	12
15	15	10
16	5	6
17	3	4

Ans: Standard deviation for boys

$x = 15$

x	f	fx	$d = x - A$	d^2	fd	fd^2
13	12	156	−2	4	−24	48
14	15	210	−1	1	−15	15
15	15	225	0	0	0	0
16	5	80	1	1	5	5
17	3	51	2	4	6	12
	$n=50$	$\Sigma fx = 722$			-28	80

$$\bar{x} = \frac{722}{50} = 14.44 \cong 15$$

$$\sigma = \sqrt{\frac{\Sigma fd^2}{n} - \left(\frac{\Sigma fd}{n}\right)^2} = \sqrt{\frac{80}{50} - \left(\frac{-28}{50}\right)^2} = \sqrt{1.6 - 0.3136} = 1.1342$$

$$C.V = \frac{\sigma}{\bar{x}} \times 100 = \frac{1.1342}{14.44} \times 100 = 7.8545\%$$

standard deviation for girls.

x	f	fx	$d = x - A$	d^2	fd	fd^2
13	18	234	−2	4	−36	72
14	12	168	−1	1	−12	12
15	10	150	0	0	0	0
16	6	96	1	1	6	6
17	4	68	2	4	08	16
	$n=40$	$\Sigma fx = 716$			-34	106
		17.9				

$$\bar{x} = \frac{716}{40} = 17.9 \cong 15$$

$$\sigma = \sqrt{\frac{\Sigma fd^2}{n} - \left(\frac{\Sigma fd}{n}\right)^2} = \sqrt{\frac{106}{40} - \left(\frac{-34}{40}\right)^2} = \sqrt{2.65 - 0.7225} = 1.3883.$$

$$\therefore C.V = \frac{\sigma}{\bar{x}} \times 100 = \frac{1.3883}{14.32} \times 100 = 9.6948$$

(5) Calculate First & third quartile (May 2017)

X	No. of persons	C.F	
250	20	20	
Q1 300	14	34	← Q1
325	6	40	
350	26	66	
375	9	75	
Q3 400	13	88	
600	4	92	
	n = 92		

Ans:

$$Q_1 = \frac{n}{4} = \frac{92}{4} = 23^{rd} \text{ term}$$

$Q_1 = 300$

$$Q_3 = 3n/4 = \frac{3(92)}{4} = 69^{th} \text{ term} \longrightarrow$$

$Q_3 = 375$

(6) An agent obtained samples of bulbs from two companies. He had tested them for durability and got the following results

Life	Type-A	Type B
17-19	100	30
19-21	160	420
21-23	260	120
23-25	80	30

which company's bulbs are more uniform life?

Ans: Standard deviation for Type A.

X	f	midvalue(x)	$d = X - A$, $X - 20$	$d' = d/C$	fd'	d'^2	fd'^2
17-19	100	18	−2	−1	−100	1	100
19-21	160	20	0	0	0	0	0
21-23	260	22	2	1	260	1	260
23-25	80	24	4	2	160	4	320
	600				320		680

$$\bar{x} = A + \frac{\Sigma fd'}{n} \times c$$

$$= 20 + \frac{320}{600} \times 2 = 21.0667 \simeq 20$$

$$\sigma = \sqrt{\frac{\Sigma fd'^2}{n} - \left(\frac{\Sigma fd'}{n}\right)^2} \times c$$

$$= \sqrt{\frac{680}{600} - \left(\frac{320}{600}\right)^2} \times 2 = \sqrt{1.133 - 0.2844} \times 2$$

$$= \sqrt{0.8488} \times 2$$

$$= 1.8427$$

$$C.V = \frac{\sigma}{\bar{x}} \times 100 = \frac{1.8427}{21.0667} \times 100 = 8.7465\ \%.$$

Standard deviation for Type B

x	f	mid value (x)	$d = x - A$	$d' = \frac{d}{c}$	fd'	d'^2	fd'^2
17-19	30	18	−2	−1	−30	1	30
19-21	120	20	0	0	0	0	0
21-23	120	22	2	1	120	1	120
23-25	30	24	4	2	60	4	120
	$n = 600$				$\Sigma fd' = 150$		$\Sigma fd'^2 = 270$

$$\bar{x} = A + \frac{\Sigma fd'}{n} \times c$$

$$= 20 + \frac{150}{600} \times 2 = 20 + 0.5 = 20.5$$

$$\sigma = \sqrt{\frac{\Sigma fd'^2}{n} - \left(\frac{\Sigma fd'}{n}\right)^2} \times c = \sqrt{\frac{270}{600} - \left(\frac{150}{600}\right)^2} \times 2$$

$$= \sqrt{0.45 - (+0.25)^2} \times 2$$

$$= \sqrt{0.45 - 0.0625} \times 2$$

$$= \sqrt{0.3875} \times 2 = 1.245.$$

$$C.V = \frac{\sigma}{\bar{x}} \times 100 = \frac{1.245}{20.5} \times 100 = 6.073\ \%.$$

C.V is less in Type B. So The bulb B company is more uniform

Units. Index numbers

1) Construct the Fisher's index for the following and show whether it satisfies TRT and FRT.

Commodities		A	B	C	D	E
Base year	Price	8	2	1	2	1
	Qnty	50	15	20	10	40
current year	Price	20	6	2	5	3
	qty	26	10	25	8	30

Ans:

P_0	q_0	P_1	q_1	P_0q_1	P_0q_0	P_1q_0	P_1q_1
8	50	20	26	208	400	1000	520
2	15	6	10	20	30	90	60
1	20	2	25	25	20	40	50
2	10	5	8	16	20	50	40
1	40	3	30	30	40	120	90
				299	510	1300	760

$$P_{01} = \sqrt{\frac{\Sigma P_1q_0}{\Sigma P_0q_0} \times \frac{\Sigma P_1q_1}{\Sigma P_0q_1}} \times 100$$

$$= \sqrt{\frac{1300}{510} \times \frac{760}{299}} \times 100$$

$$= \sqrt{6.47911} \times 100 = 254.54\%$$

FRT:

$$\sqrt{\frac{\Sigma P_1q_0}{\Sigma P_0q_0} \times \frac{\Sigma P_1q_1}{\Sigma P_0q_1}} \times \sqrt{\frac{\Sigma q_1P_0 \times \Sigma q_1P_1}{\Sigma q_0P_0 \times \Sigma P_1q_0}}$$

$$\frac{760}{510} = \frac{\Sigma P_1q_1}{\Sigma P_0q_0} = \left(\frac{760}{510}\right).$$

TRT

$$\sqrt{\frac{\Sigma P_1Q_0}{\Sigma P_0q_0} \times \frac{\Sigma P_1q_1}{\Sigma P_0q_1}} \times \sqrt{\frac{\Sigma P_0q_1 \times \Sigma P_0q_0}{\Sigma P_1q_1 \times \Sigma P_1q_0}}$$

$$\sqrt{\frac{1300}{510} \times \frac{760}{299}} \times \sqrt{\frac{299}{760} \times \frac{510}{1300}}$$

$$= 1$$