

Unit 24 : chapter 5: Network analysis

1) Critical path method

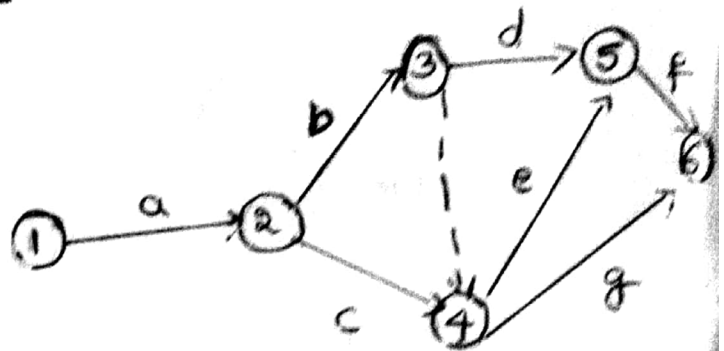
2) Project Evaluation and Review Technique (PERT)

Fulkerson's rule of numbering events:

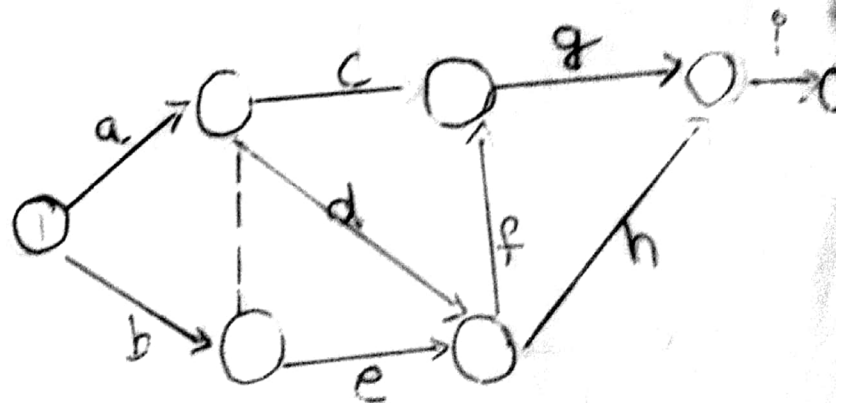
d)

| Activity | Predecessor |
|----------|-------------|
| a | - |
| b | a |
| c | a |
| d | b |
| e | b,c |
| f | d,e |
| g | b,c |

Ans:

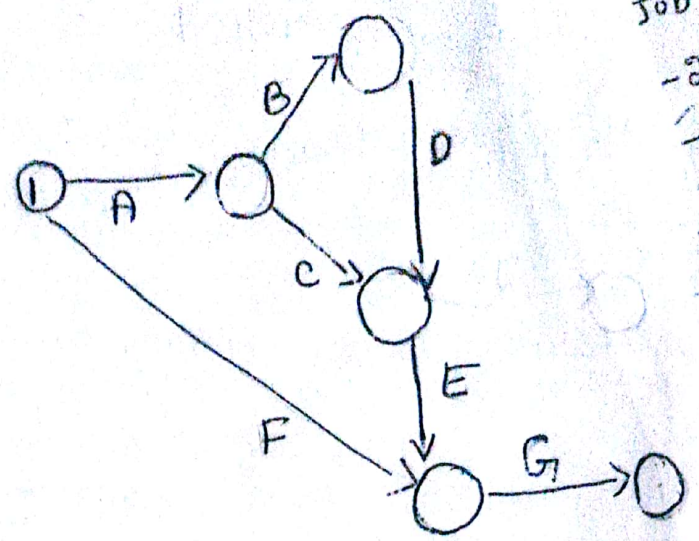


| activity | Predecessor |
|----------|-------------|
| a | - |
| b | a,b |
| e | a,b |
| d | b |
| f | d,e |
| g | c,f |
| h | d,e |
| i | g,h |



3)

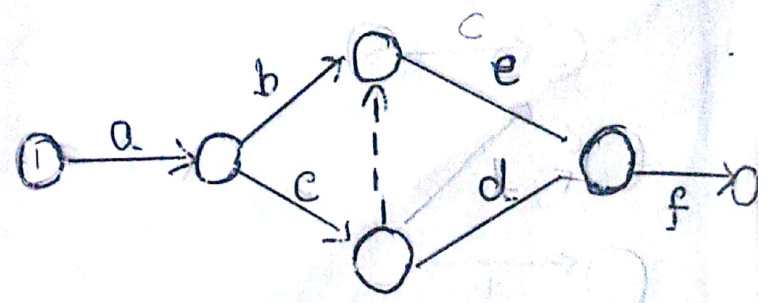
| Job | Predecessor |
|-----|-------------|
| A | - |
| B | A |
| C | A |
| D | B |
| E | C, D |
| F | - |
| G | E, F |



ritical
Job
-2
-3
-1

4)

| Job | Predecessor |
|-----|-------------|
| a | - |
| b | a |
| c | a |
| d | c |
| e | b, c |
| f | d, e |



1) Critical path method (CPM)

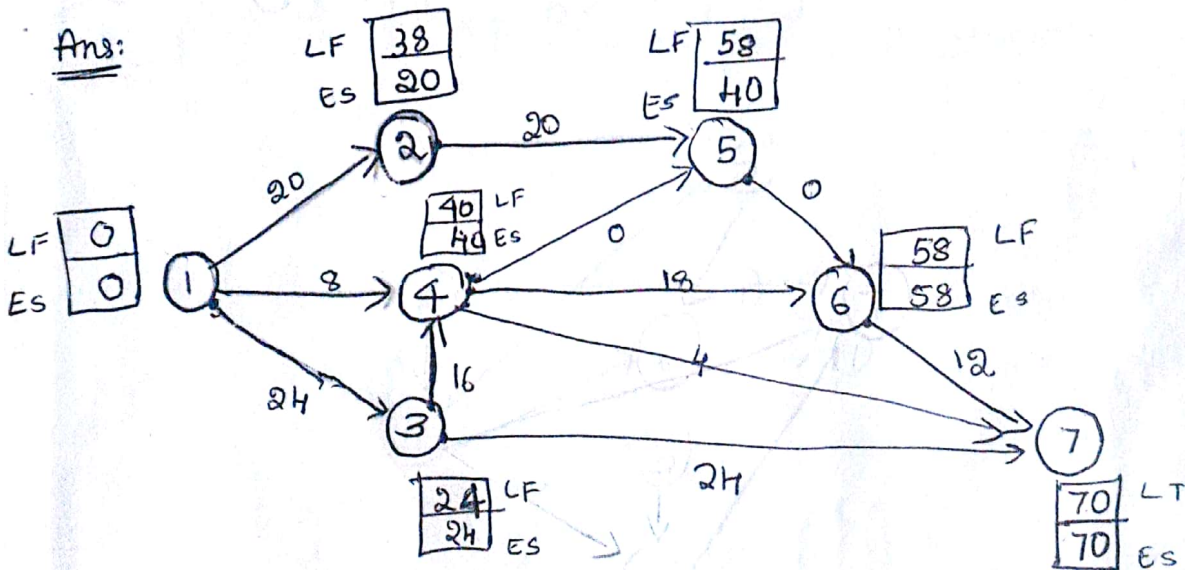
| Job | Duration |
|-----|----------|
| 1-2 | 20 |
| 1-3 | 24 |
| 1-4 | 8 |
| 2-5 | 20 |
| 3-4 | 16 |
| 3-7 | 24 |
| 4-5 | 0 |
| 4-6 | 18 |
| 5-6 | 0 |
| 4-7 | 4 |
| 6-7 | 12 |

a) Draw the network

b) Determine the critical path

c) compute ES, EF, LS, LF, TS, FS

Ans:



ES

LF

① $0+8=8$ $24+16=40$

⑥ $70-12=58$

② $58-20=38$

⑤ $20+20=40$

⑦ $58-0=58$

③ $38-0=38$

④ $0+16=16$ $18+0=18$

④ $40-0=40$
 $58-18=40$
 $70-4=66$

$40-8=32$

⑦ $24+24=48$

$40+4=44$

$58-12=46$

③ $70-24=46$
 $40-16=24$

$24-24=0$

| Job | | Earliest time | | Latest time | | Total slack (TS) | Free slack |
|-----|----|------------------------------|---------|-------------|----------------------|------------------|------------|
| | | start (ES) From (Network) | (EF) | LS | LF (From Network) | | |
| 1 | 2 | 3 | 4 = 2+3 | 5 = 6-2 | 6 | 7 = 5-3 | 8 |
| 1-2 | 20 | 0 | 20 | 18 | 38 | 18 | 0 |
| 1-3 | 24 | 0 | 24 | 0 | 24 | 0 | 0 |
| 1-4 | 8 | 0 | 8 | 32 | 40 | 32 | 32 |
| 2-5 | 20 | 20 | 40 | 38 | 58 | 18 | 0 |
| 3-4 | 16 | 24 | 40 | 24 | 40 | 0 | 0 |
| 3-7 | 24 | 24 | 48 | 46 | 70 | 22 | 22 |
| 4-5 | 0 | 40 | 40 | 58 | 58 | 18 | 0 |
| 4-6 | 18 | 40 | 58 | 40 | 58 | 0 | 0 |
| 5-6 | 0 | 40 | 40 | 58 | 58 | 18 | 18 |
| 4-7 | 4 | 40 | 44 | 66 | 70 | 26 | 26 |
| 6-7 | 12 | 58 | 70 | 58 | 70 | 0 | 0 |

Project Evaluation and Review Technique (PERT)

Optimistic time (t_o): It is the shortest possible time taken to complete the activity if everything goes well.

Pessimistic time (t_p): It is the longest time taken by an activity to complete if everything goes badly

Most likely time (t_m): It is the estimate of normal time an activity would take

$$\text{Expected time } (t_e) : t_e = \frac{t_o + 4t_m + t_p}{6}$$

$$\text{Standard deviation } \sigma = \frac{t_p - t_o}{6}$$

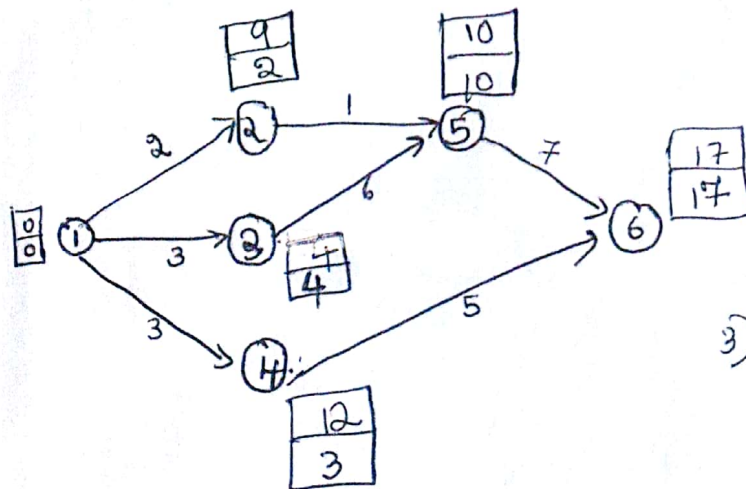
$$\text{Variance} = \sigma^2 = \left(\frac{t_p - t_o}{6} \right)^2$$

$$Z = \frac{\text{Due date} - \text{expected date of completion}}{\sqrt{\text{Variance}}}$$

| Activity | optimistic | most likely | Pessimistic |
|----------|------------|-------------|-------------|
| 1-2 | 1 | 1 | 7 |
| 1-3 | 1 | 4 | 7 |
| 1-4 | 2 | 2 | 8 |
| 2-5 | 1 | 1 | 1 |
| 3-5 | 2 | 5 | 14 |
| 4-6 | 2 | 5 | 8 |
| 5-6 | 3 | 6 | 15 |

Ans:

| Activity | t_o | t_m | t_p | $t_e = \frac{t_o + 4t_m + t_p}{6}$ | $\sigma^2 = \left(\frac{t_p - t_o}{6}\right)^2$ |
|----------|-------|-------|-------|------------------------------------|---|
| 1-2 | 1 | 1 | 7 | 2 | 1 ✓ |
| 1-3 | 1 | 4 | 7 | 3 | 1 ✓ |
| 1-4 | 2 | 2 | 8 | 3 | 0 |
| 2-5 | 1 | 1 | 1 | 1 | 4 ✓ |
| 3-5 | 2 | 5 | 14 | 6 | 1 ✓ |
| 4-6 | 2 | 5 | 8 | 5 | 1 ✓ |
| 5-6 | 3 | 6 | 15 | 7 | 4 ✓ |



3) i) Due date = $17 - 4 = 13$ week

ii) Due date = $17 + 4 = 21$ w

1) critical path $1 \rightarrow 3 \rightarrow 5 \rightarrow 6$

2) $\sigma^2 = 1 + 4 + 4 = 9$

$\sigma = 3$

$$i) z = \frac{13-17}{3} = \underline{\underline{-1.333}}$$

$$ii) z = \frac{21-17}{3} = \underline{\underline{+1.333}}$$