Least Cost Method: [matrix minima method or Lowest Cost Entry method].

Problems:

1) Solve by Least-Cost method

				Supply
2	3	11	7	6
1	0	6	1	1
5	8	15	9	10
Demand 7	5	3	2	

Ans:

2	3	11	7	6
1	[1] 0	6	1	1
5	8	15	9	10
7	5	3	2	

[6] 2	3	11	7	6
5	8	15	9	10
7	4	3	2	

[1] 5	[4] 8	[3] 15	[2] 9	10
1	4	3	2	

[6] 2	3	11	7	6
1	[1] 0	6	1	1
[1] 5	[4] 8	[3] 15	[2] 9	10
7	5	3	2	

$Z = 6(2) + 1(0) + 1(5) + 4(8) + 3(15) + 2(9)$

$= 12 + 5 + 32 + 45 + 18$

$\boxed{Z = 112}$

North West Corner Method:

1) Solve by transportation problem's method

				Supply
10	0	20	11	15
12	7	9	20	25
0	14	16	18	5
Demand 5	15	15	10	

Ans:

[5] 10	[10] 0	20	11	15
12	[5] 7	[15] 9	[5] 20	25
0	14	16	[5] 18	5
5	15	15	10	

$= 5(10) + 10(0) + 5(7) + 15(9) + 5(20) + 5(18)$

$= 50 + 0 + 35 + 135 + 100 + 90$

$Z = 410$

2.

				Supply
18	24	28	32	10
8	13	17	18	5
10	15	19	22	15
Demand 5	10	8	7	

Ans:

[5] 18	[5] 24	28	32	10
8	[5] 13	[0] 17	18	5
10	15	[8] 19	[7] 22	15
5	10	8	7	

$Z = 5(18) + 5(24) + 5(13) + 0(17) + 8(19) + 7(22)$

$= 90 + 120 + 65 + 152 + 154$

$Z = 581$

2)

9	12	9	8	4	3	5
7	3	6	8	9	4	8
4	5	6	8	10	14	6
7	3	5	7	10	9	7
2	3	8	10	2	4	3
3	4	5	7	6	4	

Ans:

9	12	9	8	4	3	5
7	3	6	8	9	4	8
4	5	6	8	10	14	6
7	3	5	7	10	9	7
[3] 2	3	8	10	2	4	~~3~~
~~3~~	4	5	7	6	4	

12	9	8	4	3	5
[4] 3	6	8	9	4	~~8~~ 4
5	6	8	10	14	6
3	5	7	10	9	7
~~4~~	5	7	6	4	

9	8	4	[4] 3	~~5~~ 1
6	8	9	4	~~4~~
6	8	10	14	6
5	7	10	9	7
5	7	6	~~4~~	

9	8	[1] 4	~~1~~
6	8	9	4
6	8	10	6
5	7	10	7
5	7	~~6~~ 5	

6	8	9	4
6	8	10	6
[5] 5	7	10	~~7~~ 2
~~5~~	7	5	

8	9	4
8	10	6
[2] 7	10	2
~~7~~ 5	5	

8	9	4
[5] 8	10	~~6~~ 1
~~5~~	5	

[4] 9	4
[1] 10	1
5	

9	12	9	8	[1] 4	[4] 3	5
7	[4] 3	6	8	[4] 9	4	6
4	5	6	[5] 8	[1] 10	14	6
7	3	[5] 5	[2] 7	10	9	7
2 [3]	3	8	10	2	4	3
3	4	5	7	6	4	

$Z = 1(4) + 4(3) + 4(3) + 4(9) + 5(8) + 1(10) + 5(5) + 2(7) + 3(2)$

$= 4 + 12 + 12 + 36 + 40 + 10 + 25 + 14 + 6$

$Z = 159$

VOGEL'S APPROXIMATION METHOD (VAM)

or (UNIT COST PENALTY METHOD)

1.

19	30	50	10	7
70	30	40	60	9
40	8	70	20	18
5	8	7	14	

ns:

19	30	50	10	7	19 - 10 = 9
70	30	40	60	9	40 - 30 = 10
40	[8] 8	70	20	~~18~~ 10	20 - 8 = 12
5	~~8~~	7	14		
40-19 = 21	30-8 = 22 ↑	50-40 = 10	20-10 = 10		

[5] 19	50	10	~~7~~ 2	19 - 10 = 10
70	40	60	9	60 - 40 = 20
40	70	20	10	40 - 20 = 20
~~5~~	7	14		
40-19 = 21 ↑	50-40 = 10	20-10 = 10		

50	10	2	50-10 = 40
40	60	9	60 - 40 = 20
70	[10] 20	10	70 - 20 = 50 ←
7	~~14~~ 4		
50 - 40 = 10	20 - 10 = 10		

50	[2] 10	~~2~~	50 - 10 = 40
[7] 40	[2] 60	9	60 - 40 = 20
7	~~4~~ 2		
50-40 = 10	60-10 = 50		

[5] 19	30	50	[2] 10	7
70	30	[7] 40	[2] 60	9
40	[8] 8	70	[10] 20	18
5	8	7	14	

$Z = 5(19) + 2(10) + 7(40) + 2(60) + 8(8) + 10(20)$

$= 95 + 20 + 280 + 120 + 64 + 200$

$Z = 779$

MODIFIED DISTRIBUTION METHOD [MODI]

					Capacity
	19	30	50	10	7
	70	30	40	60	9
	40	8	70	20	18
Requirements	5	8	7	14	

	v_1	v_2	v_3	v_4	
u_1	[5] 19	30	50	[2] 10	7
u_2	70	30	[7] 40	[2] 60	9
u_3	40	[8] 8	70	[10] 20	18
	5	8	7	14	

Occupied cells:

$u_i + v_j = C_{ij}$

$u_1 + v_1 = 19 \qquad u_1 = 0 \qquad v_1 = 19$

$u_1 + v_4 = 10 \qquad u_1 = 0 \qquad v_4 = 10$

Scanned by CamScanner

$u_2+v_3=40 \quad u_2=50 \quad v_3=-10$

$u_2+v_4=60 \quad u_2=50 \quad v_4=10$

$u_3+v_2=8 \quad u_3=10 \quad v_2=-2$

$u_3+v_4=20 \quad u_3=10 \quad v_4=10$

Unoccupied Cells: $\Delta_{ij}=C_{ij}-(u_i+v_j)$

$\Delta_{12}=C_{12}-(u_1+v_2)=30-(0-2)=32$

$\Delta_{13}=C_{13}-(u_1+v_3)=50-(0-10)=60$

$\Delta_{21}=C_{21}-(u_2+v_1)=70-(50+19)=1$

$\Delta_{22}=C_{22}-(u_2+v_2)=30-(50-2)=-18$

$\Delta_{31}=C_{31}-(u_3+v_1)=40-(10+19)=11$

$\Delta_{33}=C_{33}-(u_3+v_3)=70-(10-10)=70$

[5] 19	30	50	[2] 10
70	[0] $+\theta$ 30	[7] 40	[2] $-\theta$ 60
40	[8] $-\theta$ 8	70	[10] $+\theta$ 20

$\min(2-\theta,\ 8-\theta)$

$\theta=2$

[5] 19	30	50	[2] 10
70	[2] 30	[7] 40	[0] 60
40	[6] 8	70	[12] 20

$=5(19)+2(10)+2(30)+7(40)+0(60)+6(8)+12(20)$

$=95+20+60+280+48+240$

$Z=743$

Algebraic method (solution of 2×2 games)

① Player B

Player A		I (q)	II (1-q)	row min
	p	8	-3	-3
	1-p	-3	1	-3
Coloumn max		8	1	

maximin ($\underline{V}$)

minimax ($\bar{V}$)

maximum of row = {-3, -3} = -3. $\underline{V} \neq \bar{V}$.

minimum of column = {8, 1} = 1.

∴ No saddle point.

a = 8 b = -3 c = -3 d = 1

$$p = \frac{d-c}{(a+d)-(b+c)} = \frac{1-(-3)}{(8+1)-(-3-3)} = \frac{4}{9+6} = \frac{4}{15}$$

$$1-p = 1 - \frac{4}{15} = \frac{11}{15}$$

$$q = \frac{d-b}{(a+d)-(b+c)} = \frac{1-(-3)}{(a+d)-(b+c)} = \frac{4}{9+6} = \frac{4}{15}$$

$$1-q = 1 - \frac{4}{15} = \frac{11}{15}.$$

value of the game $V = \frac{ad-bc}{(a+d)-(b+c)} = \frac{8(1)-(-3)(-3)}{15} = \frac{-1}{15}$.

[op]timum strategy for Player A ⇒ $\left(\frac{4}{15}, \frac{11}{15}\right)$

[op]timum strategy for Player B ⇒ $\left(\frac{4}{15}, \frac{11}{15}\right)$

$= \frac{-1}{15}$ → the game is unfair of A.

Games with saddle points

① Solve

	B_1	B_2	B_3	B_4
A_1	-5	2	0	7
A_2	5	6	4	8
A_3	4	0	2	-3

Ans:

					row min
	-5	2	0	7	-5
	5	6	4	8	(4) maximin ($\underline{v}$)
	4	0	2	-3	-3
column max	5	6	(4)	8	

minimax ($\bar{v}$)

maximum of row = {-5, 4, -3} = 4.

minimum of column = {5, 6, 4, 8} = 4.

∴ saddlepoint = 4 = value of the game

② B

	I	II	III	IV	V
A I	9	3	1	8	0
A II	6	5	4	6	7
A III	2	4	4	3	8
A IV	5	6	2	2	1

Ans:

						row min
	9	3	1	8	0	0
	6	5	4	6	7	(4) maximin ($\underline{v}$)
	2	4	4	3	8	2
	5	6	2	2	1	1
column max	9	6	(4)	8	8	

minimax ($\bar{v}$)

maximum of row = {0, 4, 2, 1} = 4

minimum of column = {9, 6, 4, 8, 8} = 4.

∴ saddle point = 4 = value of the game

Dominance method

A

	1	2	3	4	5	row min
B 1	6	15	30	21	6	
2	3	3	6	6	4	
3	12	12	24	36	3.	
column max						

2nd row elements are less than 1st row. so row is dominated. and can be removed

	1	2	3	4	5
1	6	15	30	21	6
3	12	12	24	36	3

	1	3	4	5
1	6	30	21	6
3	12	24	36	3

column 2 is greater than (or equal to) column I and removed

column 3 is greater than column I. and removed

column 4 is greater than column 5. and removed

		q 1	(1-q) 5	row min
A	p 1	a 6	b 6	6
	(1-p) 3	c 12	d 3	(3)
	column max	(12)	6	

min of row = {6, 3} = 3

max of column = {12, 6} = 12.

maximin ≠ minmax.

∴ No saddle point.

$$p = \frac{d-c}{(a+d)-(b+c)} = \frac{3-12}{(6+3)-(6+12)} = \frac{-9}{9-18} = \frac{-9}{-9} = 1$$

$$1-p = 1-1 = 0$$

$$q = \frac{d-b}{(a+d)-(b+c)} = \frac{3-6}{-9} = \frac{-3}{-9} = 1/3$$

$$1-q = 1-\frac{1}{3} = \frac{2}{3}.$$

Value of the game $= \frac{ad-bc}{(a+d)-(b+c)} = \frac{6(3)-6(12)}{(6+3)-(6+12)}$

$= \frac{-54}{-9} = 6.$

Optimum strategy for player A $\Rightarrow (x_1, x_2, x_3)$

$\Rightarrow (1, 0, 0).$

Optimum strategy for player B $\Rightarrow (y_1\ y_2\ y_3\ y_4\ y_5)$

$\Rightarrow \left(\frac{1}{3}, 0, 0, 0, \frac{2}{3}\right).$

2)

		y_1 B1	y_2 B2	y_3 B3	y_4 B4	y_5 B5	row min
x_1	A1	~~4~~	~~4~~	~~2~~	~~4~~	~~6~~	
x_2	A2	8	6	8	−4	0	
x_3	A3	10	2	4	10	12	
Column max.							

Ans: row I is minimum than row II and removed

	B1	B2	B3	B4	B5
A2	8	6	8	−4	0
A3	10	2	4	10	12

column I is greater than column II and removed

column III is greater than column II and removed.

column V is greater than column IV and removed.

	B2 q	B4 (1−q)
A2 p	6	−4
A3 (1−p)	2	10

$$P = \frac{d-c}{(a+d)-(b+c)} = \frac{10-2}{(6+10)-(-4+2)} = \frac{8}{16+2} = \frac{8}{18} = \frac{4}{9}$$

$$1-P = 1-\frac{4}{9} = \frac{5}{9}.$$

$$q = \frac{d-b}{(a+d)-(b+c)} = \frac{10-(-4)}{(6+10)-(-4+2)} = \frac{14}{18} = \frac{7}{9}.$$

$$1-q = 1-\frac{7}{9} = \frac{2}{9}.$$

$$V = \frac{ad-bc}{(a+d)-(b+c)} = \frac{6(10)-(-4)(2)}{18} = \frac{68}{18} = \frac{34}{9}.$$

optimum strategy for $A = (x_1, x_2, x_3) = (0, 4/9, 5/9)$

optimum strategy for $B = (y_1, y_2, y_3, y_4, y_5)$
$= (0, 7/9, 0, 2/9, 0)$.

$$V = \frac{34}{9}.$$

Graphical method for (2×n) or (m×2) games.

1) Use graphical method to reduce the following game and hence solve.

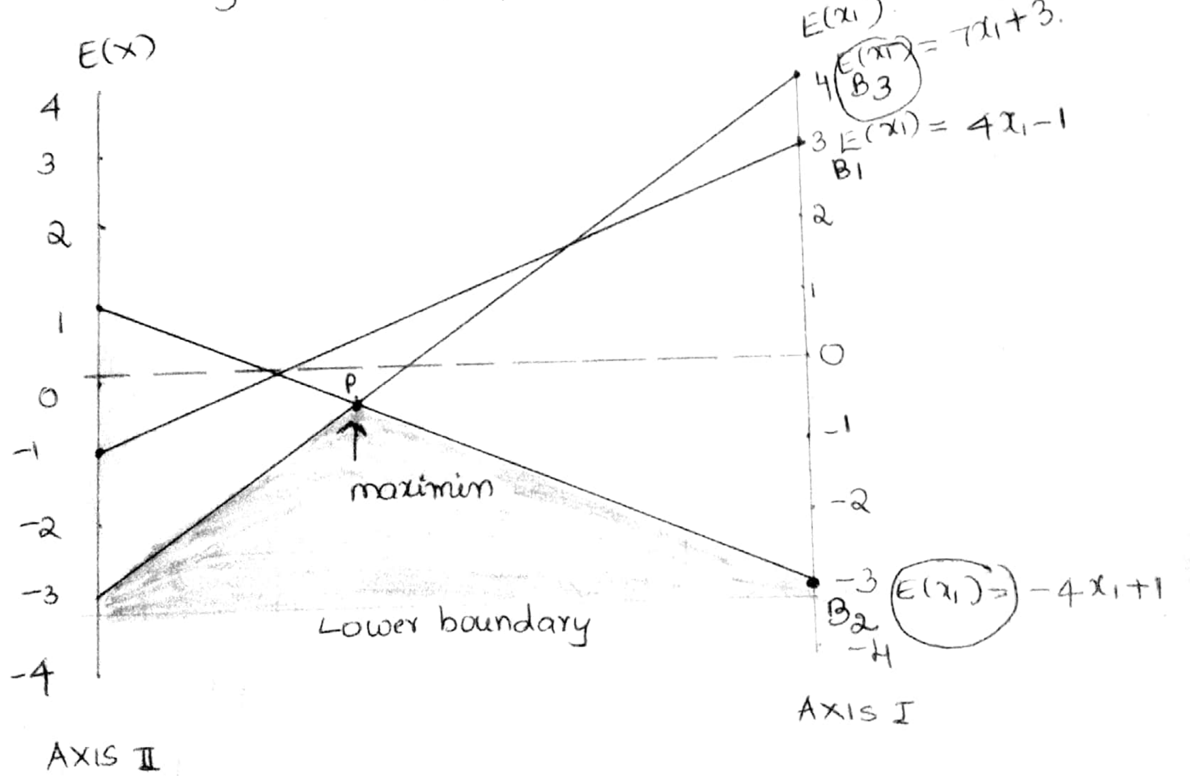
		y_1 B_1	B y_2 B_2	y_3 B_3
x_1	A A_1	3	−3	4
$(1-x_1)$	A_2	−1	1	−3

maximin

minimax

Ans:

B's strategy	A's payoff $E(x_1)$
B_1	$E(x_1) = 3x_1 + (-1)(1-x_1) = 3x_1 - 1 + x_1 = 4x_1 - 1$
B_2	$E(x_1) = -3x_1 + (1-x_1)(1) = -4x_1 + 1$
B_3	$E(x_1) = 4x_1 + (1-x_1)(-3) = 7x_1 + 3.$



	B_2	B_3
A_1	−3	4
A_2	1	−3

$$x_1 = \frac{d-c}{(a+d)-(b+c)} = \frac{-3-1}{(-3-3)-(4+1)} = \frac{-4}{-11} = \frac{4}{11}$$

$$1-x_1 = 1-\frac{4}{11} = \frac{7}{11}$$

$$y_2 = \frac{d-b}{(a+d)-(b+c)} = \frac{-3-4}{-11} = \frac{7}{11}$$

$$1-y_2 = 1-\frac{7}{11} = \frac{4}{11}$$

$$V = \frac{ad-bc}{(a+d)-(b+c)} = \frac{(-3)(-3)-(4)(1)}{-11} = -\frac{5}{11}$$

optimal strategy for $A = (x_1\ x_2) = (4/11, 7/11)$

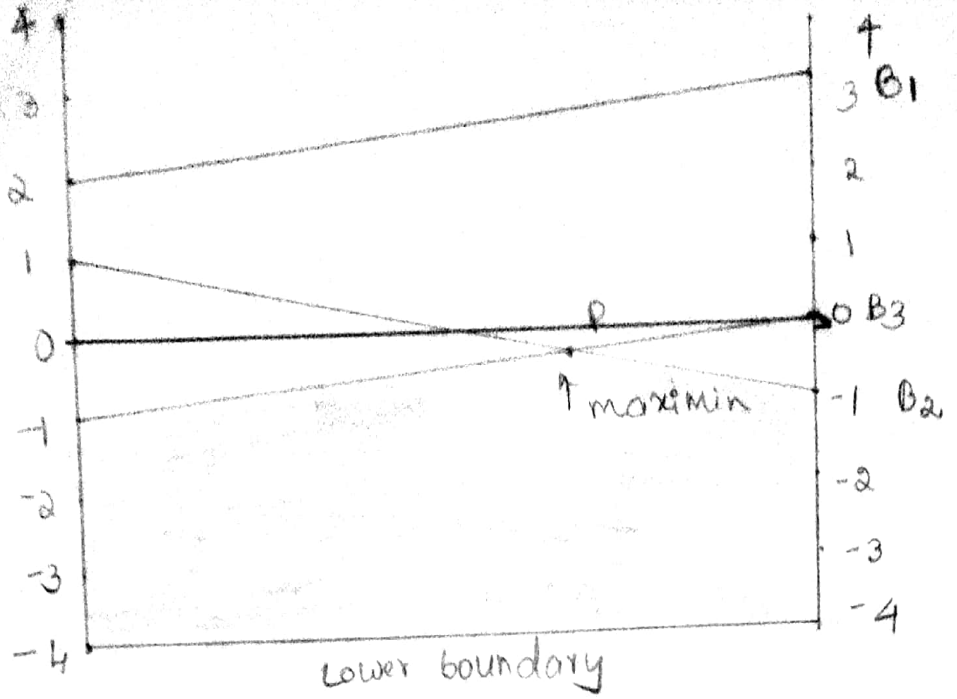
optimal strategy for $B = (y_1, y_2, y_3) = 0, 7/11, 4/11)$.

$V = -5/11$

2)

		B_1 y_1	B_2 y_2	B_3 y_3
A_1	x_1	3	-1	0
A_2	$x_2 (1-x_1)$	2	1	-1

B's strategy	A's payoff
B_1	$E(x_1) = 3x_1 + 2(1-x_1) = x_1+2$
B_2	$E(x_1) = -x_1 + 1(1-x_1) = -2x_1+1$
B_3	$E(x_1) = 0 \cdot x_1 + (-1)(1-x_1) = x_1 - 1$



	B_2	B_3
A_1	-1	0
A_2	1	-1

$$x_1 = \frac{d-c}{(a+d)-(b+c)} = \frac{-1-1}{(-1-1)-(0+1)} = \frac{-2}{-3} = \frac{2}{3}$$

$$1-x_1 = 1-\frac{2}{3} = \frac{1}{3}.$$

$$y_2 = \frac{d-b}{(a+d)-(b+c)} = \frac{-1-0}{-3} = \frac{1}{3}$$

$$y_3 = 1-y_2 = 1-\frac{1}{3} = \frac{2}{3}.$$

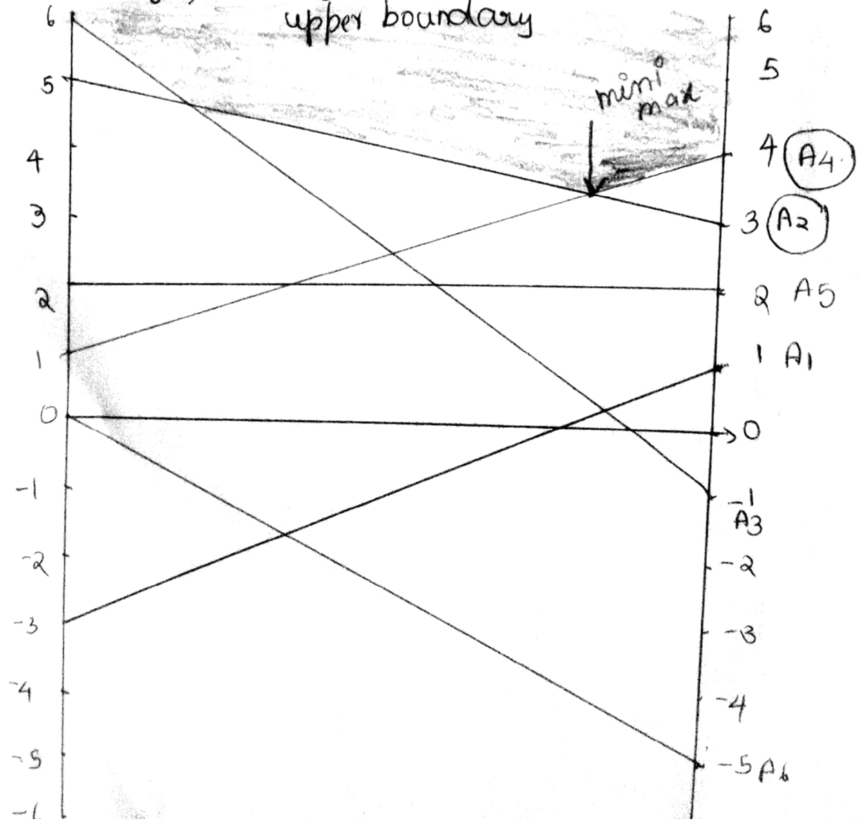
$$V = \frac{ad-bc}{(a+d)-(b+c)} = \frac{(-1)(-1)-0}{-3} = \frac{-1}{3}$$

Graphical solution of (mx2) games:

(1)

	B_1 (y_1)	B_2 ($1-y_1$)
A_1	1	-3
A_2	3	5
A_3	-1	6
A_4	4	1
A_5	2	2
A_6	-5	0

A' strategy	B's playoff
A_1	$E(y_1) = 1(y_1) + (-3)(1-y_1) = y_1 - 3 + 3y_1 = 4y_1 - 3$
A_2	$E(y_2) = 3y_1 + 5(1-y_1) = 3y_1 + 5 - 5y_1 = -2y_1 + 5$
A_3	$E(y_3) = -1y_1 + 6(1-y_1) = -7y_1 + 6$
A_4	$E(y_4) = 4y_1 + 1(1-y_1) = 3y_1 + 1$
A_5	$E(y_5) = 2y_1 + 2(1-y_1) = 2$
A_6	$E(y_6) = -5y_1 + 0(1-y_1) = -5y_1$



		B_1	B_2
		y_1	$1-y_1=y_2$
A_2	x_2	3	5
A_4	x_4 $1-x_2=$	4	1

$$y_1 = \frac{d-b}{(a+d)-(b+c)} = \frac{1-5}{4-(9)} = \frac{-4}{-5} = \frac{4}{5}$$

$$y_2 = 1-y_1 = 1-\frac{4}{5} = \frac{1}{5}$$

$$x_2 = \frac{d-c}{(a+d)-(b+c)} = \frac{1-4}{4-(9)} = \frac{3}{5}$$

$$x_4 = 1-x_2 = 1-\frac{3}{5} = \frac{2}{5}$$

$$V = \frac{ad-bc}{(a+d)-(b+c)} = \frac{3-20}{4-(9)} = \frac{+17}{5}$$

optimal strategy for A $= (x_1, x_2, x_3, x_4, x_5, x_6)$
$= (0, \frac{3}{5}, 0, \frac{2}{5}, 0, 0)$

optimal straligy for B $= (y_1, y_2) = (1/5, 3/5)$